MATH 118: Midterm 1

Name: key

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60

1. Short answer questions:

   (a) Suppose you write

   $$(x+y)^2z^2 = x^2 + y^2z^2$$

   What are the two errors you made?

   ① x and y are terms. can only manipulate exponents (exponent laws) across factors.

   ② everything to the left of z^2 should be encapsulated in parentheses since you are multiplying z^2 into ≥ 2 terms

   (b) True or false: We can simplify $\frac{x^2+x-2}{x-1}$ by crossing out the x's to become $\frac{x^2-2}{-1}$. If not, properly simplify the expression.

   False; x is both a term in the context of the entire numerator and denominator.

   $$\frac{x^2+x-2}{x-1} = \frac{(x-1)\cdot(x+2)}{(x-1)} = x+2$$

   (c) If $f(x) = \frac{x}{1-x}$, find $f(x^2-1)$.

   $$f(x^2-1) = \frac{x^2-1}{1-(x^2-1)} = \frac{x^2-1}{1-x^2+1} = \frac{x^2-1}{2-x^2}$$

   (d) If $i^2 = -1$, what is i^{531}?

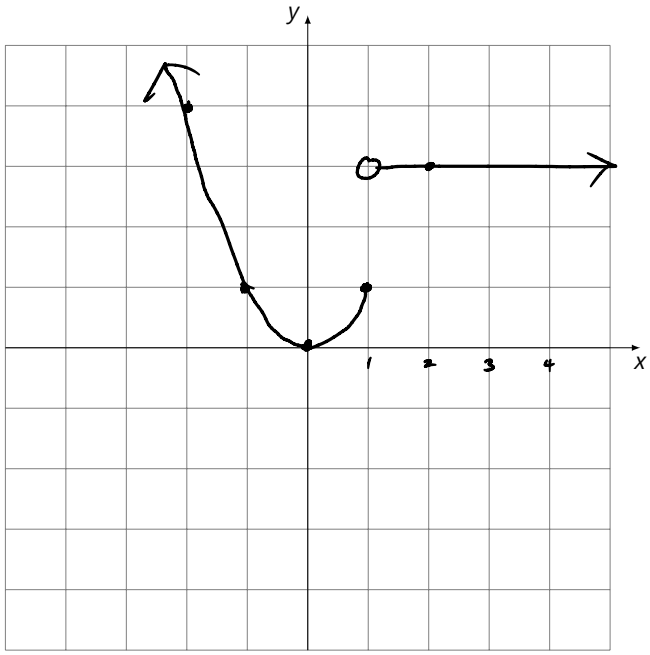
   $$i^{531} = i^{530+1}$$
   $$= i^{530}\cdot i^1$$
   $$= (i^2)^{265}\cdot i$$
   $$= (-1)^{265}\cdot i$$
   $$= -i$$

2. Suppose

$$f(x) = \begin{cases} 3 & x > 1 \\ x^2 & x \leq 1 \end{cases}$$

(a) Sketch a graph of $f(x)$.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	3



(b) What is $f(1)$?

$f(1) = 1^2 = 1$

3. Fully simplify the following using relevant properties and laws.

(a) $\left(\frac{4x^2y}{5z^{-1}}\right)^2 \cdot \frac{1}{x^2z^2} = \left(\frac{4x^2yz}{5}\right)^2 \cdot \frac{1}{x^2z^2}$

$= \frac{(4x^2yz)^2}{5^2} \cdot \frac{1}{x^2z^2}$

$= \frac{4^2 \cdot (x^2)^2 \cdot y^2 \cdot z^2}{25} \cdot \frac{1}{x^2z^2}$

$= \frac{16x^4y^2z^2}{25x^2z^2} = \boxed{\frac{16x^2y^2}{25}}$

$A^2 - B^2$, terms

(b) $\left(\frac{1}{x^2-1} - \frac{2}{x-1}\right)^2$

subtraction of fractional expressions. Find LCD. Need factors.

factors.

$= \left(\frac{1}{(x-1)(x+1)} - \frac{2}{x-1}\right)^2$

$= \left(\frac{1}{(x-1)(x+1)} - \frac{2}{x-1} \cdot \frac{(x+1)}{(x+1)}\right)^2$

$= \left(\frac{1}{(x-1)(x+1)} - \frac{2(x+1)}{(x-1)(x+1)}\right)^2$

$= \left(\frac{1-2x-2}{(x-1)(x+1)}\right)^2$

$= \left(\frac{1-2x}{(x-1)(x+1)}\right)^2$

$= \frac{(1-2x)^2}{((x-1)(x+1))^2}$

$= \boxed{\frac{(1-2x)^2}{(x-1)^2(x+1)^2}}$

4. Given $ax - bx(c + d) - ex = gx$, isolate x.

$$ax - bcx - bdx - ex = gx$$

$$ax - bcx - bdx - ex - gx = 0$$

$$x \cdot (a - bc - bd - e - g) = 0$$

$$\frac{x \cdot (a - bc - bd - e - g)}{a - bc - bd - e - g} = \frac{0}{a - bc - bd - e - g}$$

$$\boxed{x = 0}$$

5. Solve for x. Check your work if necessary.

$$x+1=\sqrt{5-x}$$

$(A+B)^2$

$(x+1)^2=(\sqrt{5-x})^2$

$x^2+2x+1=5-x$

$x^2+3x-4=0$

1 4

1 -1

$(x+4)(x-1)=0$

$x+4=0$, $x-1=0$

$x=-4$, $x=1$

check $x=-4$

$-4+1=\sqrt{5-(-4)}$

$-3=\sqrt{9}$

$-3=3$

extraneous

check $x=1$

$1+1=\sqrt{5-1}$

$2=\sqrt{4}$

$2=2$ ✓

Solution

$x=1$

6. Fully factor and simplify

$$(x^3+x^2+x+1)^2-2(x^3+x^2+x+1)+1$$

Let $y = x^3+x^2+x+1$. Substituting:

$$y^2-2y+1$$

$$=(y-1)^2$$

$$=(x^3+x^2+x+1-1)^2$$

$$=\boxed{(x^3+x^2+x)^2}$$